Using DEB theory at the population scale to limit the risk of structural sensitivity

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DEB symposium in Tromsø, Norway

Modelling a process (e.g. predation)

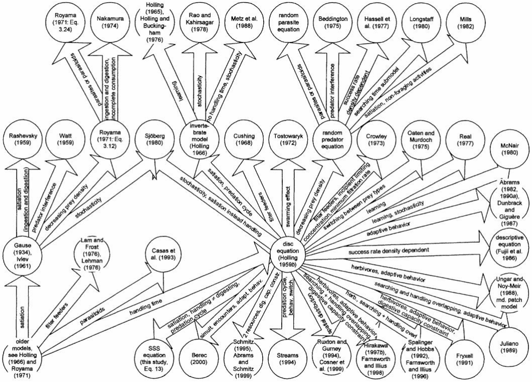
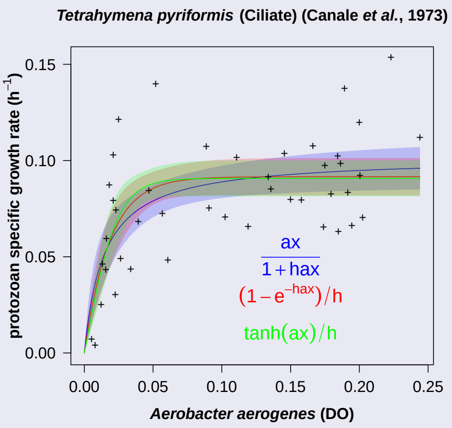


FIG. 1. A "family tree" of functional response models.

(Jeschke et al., 2002)

find / attack, handling, digestion / metabolism, spatial heterogeneity, individual variability, collective behaviour, ...

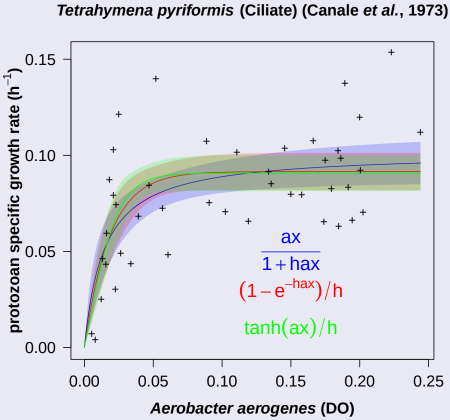
Modelling predation at the population scale



functional response

- prey eaten / predator / time

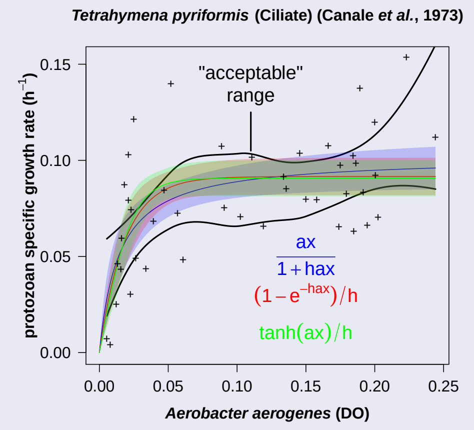
Modelling predation at the population scale



functional response

- prey eaten / predator / time
- 3 functions with the same mathematical properties ⇒ same hypotheses on **process shape**
- different hypotheses on **underlying mechanisms**

Modelling predation at the population scale



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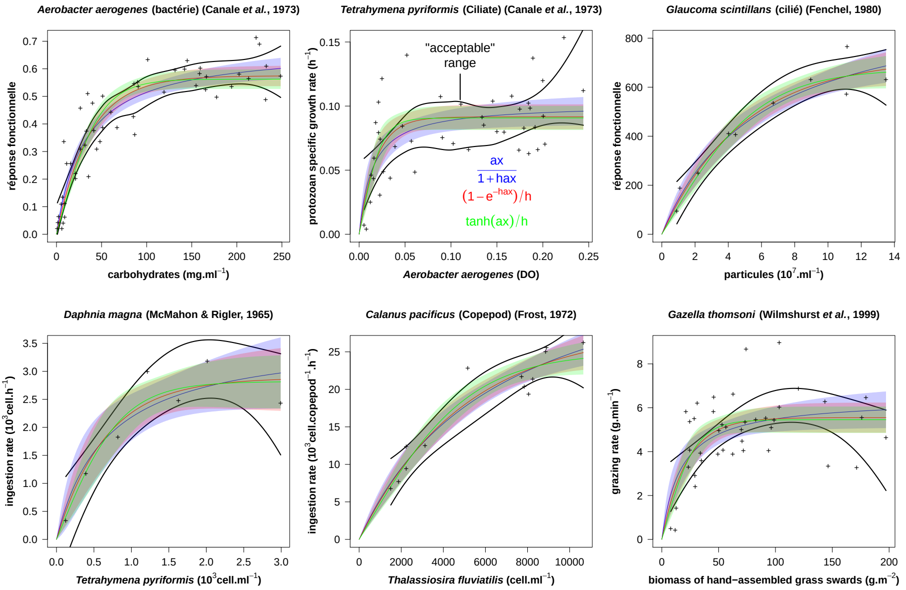
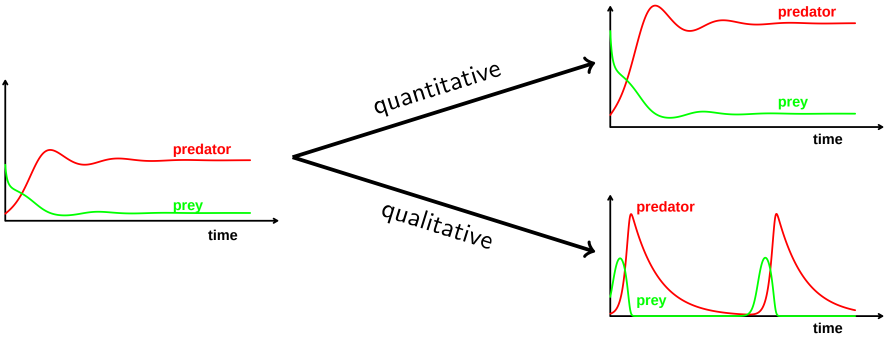


Different model... different predictions

- change in numerical values (equilibrium, period of oscillations)
- change on existence and stability of invariant sets (bifurcation) *(Kuznetsov, 2004)*



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structural sensitivity

- functions with the same mathematical properties, that fit data and have theoretical support $\Rightarrow$ different predictions

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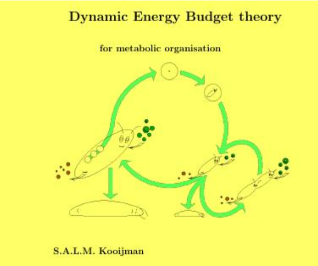
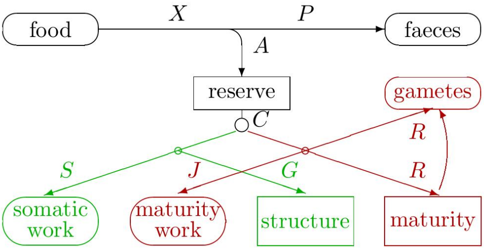
structural sensitivity

- functions with the same mathematical properties, that fit data and have theoretical support $\Rightarrow$ different predictions
- previous studies : **functional response** in predator-prey model, pathogen infection, colimited uptake of nutrients *(Myerscough et al., 1996, Wood & Thomas, 1999, Gross et al., 2004, Fussmann & Blasius, 2005, Anderson et al., 2010, Poggiale et al.,2010, Cordoleani et al., 2011)*

simple assumptions on individual metabolism

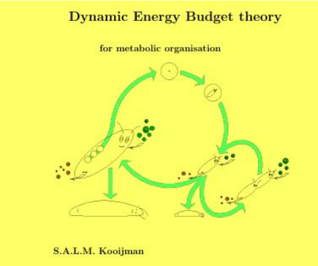
An interesting framework : DEB theory

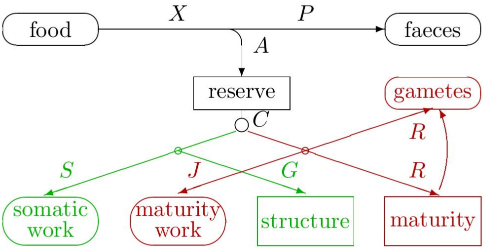




Dynamic Energy Budget theory

An interesting framework : DEB theory





Dynamic Energy Budget theory

- focus on **individual** and **energy**, mechanistic hypotheses on metabolism
- energy allocation scheme is common to most species

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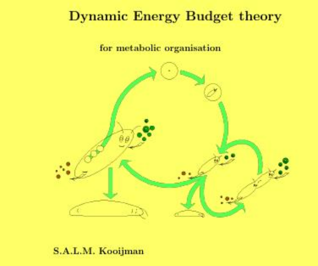
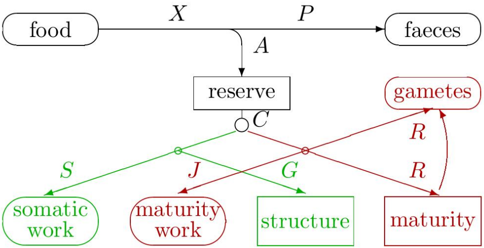
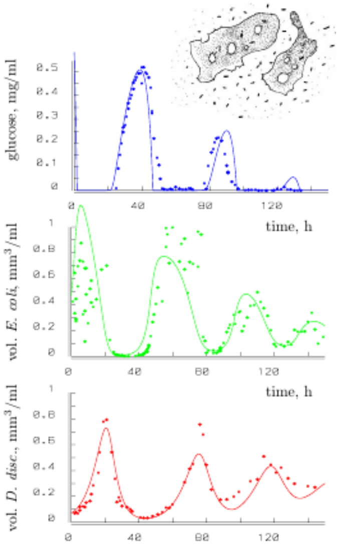




Dynamic Energy Budget theory

- focus on **individual** and **energy**, mechanistic hypotheses on metabolism
- energy allocation scheme is common to most species
- framework to build ± detailled models **consistent** between each others

A useful example : dividing unicellular organisms

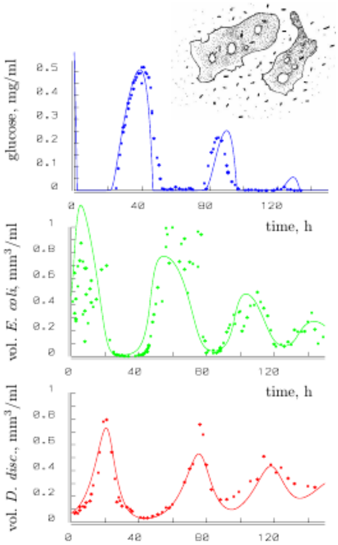


(Dent et al., 1976 ; Kooi & Kooijman, 1994 ; Kooijman, 2010)

- individual ⇒ population
- reproduction easier to model

"DEB", 5 var.

A useful example : dividing unicellular organisms



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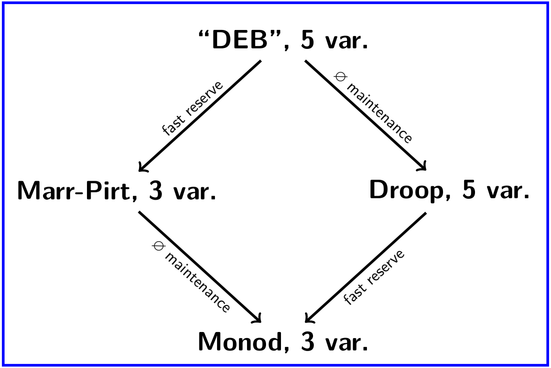


∈ DEB theory

(Dent et al., 1976 ; Kooi & Kooijman, 1994 ; Kooijman, 2010)

Different chemostat models : "DEB"

$$\begin{cases} \dfrac{dX_0}{dt} = \dot{h}(X_r - X_0) \end{cases}$$

$\downarrow$ resource $\longrightarrow$

"DEB", 5 var.

Different chemostat models : “DEB”

$$
\begin{cases}
\dfrac{dX_0}{dt} = \dot{h}(X_r - X_0) - f_1(X_0) j^1_{XAm} X_1 \\
\dfrac{de_1}{dt} = \dot{k}^1_E \left(f_1(X_0) - e_1 \right)
\end{cases}
$$

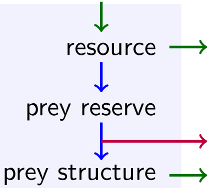
resource

prey reserve

“DEB”, 5 var.

Different chemostat models : "DEB"

$$\begin{cases} \dfrac{dX_0}{dt} = \dot{h}(X_r - X_0) - f_1(X_0) j^1_{XAm} X_1 \\ \dfrac{de_1}{dt} = \dot{k}^1_E \left(f_1(X_0) - e_1 \right) \\ \dfrac{dX_1}{dt} = \left(\dfrac{\dot{k}^1_E e_1 - \dot{k}^1_M g_1}{e_1 + g_1} - \dot{h} \right) X_1 \end{cases}$$



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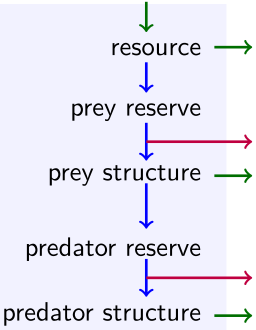
\end{cases}
$$



“DEB”, 5 var.

Different chemostat models : Droop

$$
\left\{
\begin{array}{lcl}
\dfrac{dX_0}{dt} & = & \dot{h}(X_r - X_0) - f_1(X_0) j^1_{XAm} X_1 \\
\dfrac{de_1}{dt} & = & \dot{k}^1_E \left(f_1(X_0) - e_1 \right) \\
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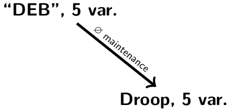
\dfrac{de_2}{dt} & = & \dot{k}^2_E \left(f_2(X_1) - e_2 \right) \\
\dfrac{dX_2}{dt} & = & \left(\dfrac{\dot{k}^2_E e_2}{e_2 + g_2} - \dot{h} \right) X_2
\end{array}
\right.
$$

↓ resource →
↓ prey reserve
↓ prey structure →
↓ predator reserve
↓ predator structure →

$\dot{k}^i_M = 0$



Different chemostat models : Marr-Pirt

$$
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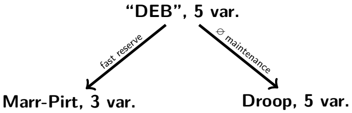
\end{cases}
$$

resource
prey reserve
prey structure
predator reserve
predator structure



$$\dot{k}^i_E, g_i \to +\infty, \frac{\dot{k}^i_E}{g_i} = \dot{\mu}_i$$

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$$
\begin{cases}
\dfrac{dX_0}{dt} = \dot{h}(X_r - X_0) - f_1(X_0) j^1_{XAm} X_1 \\[2ex]
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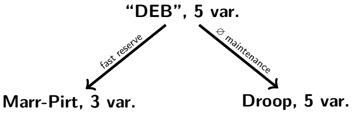
\dfrac{dX_2}{dt} = \left(\dot{\mu}_2 f_2(X_1) - \dot{k}^2_M - \dot{h} \right) X_2
\end{cases}
$$

resource

prey structure

predator structure



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Different chemostat models : Monod

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resource →

prey structure →

predator structure →

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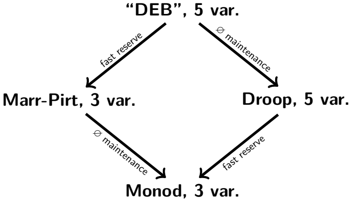
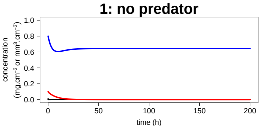
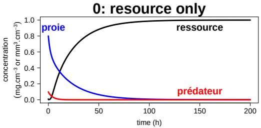
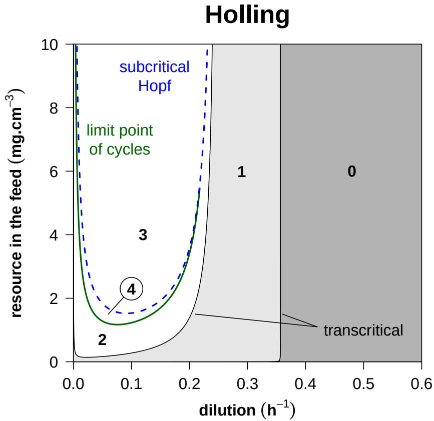


Dynamics predicted by model "DEB"







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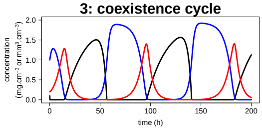
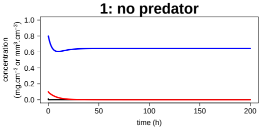
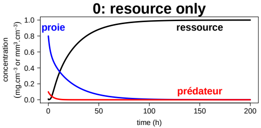
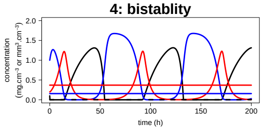
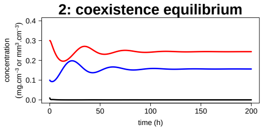
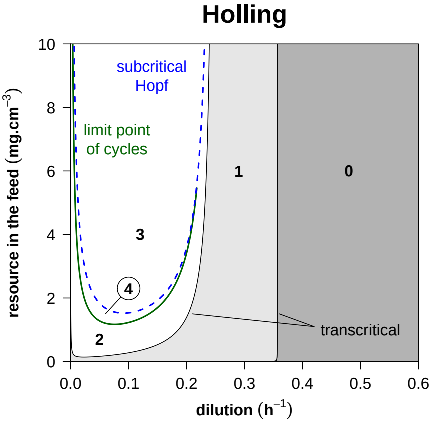




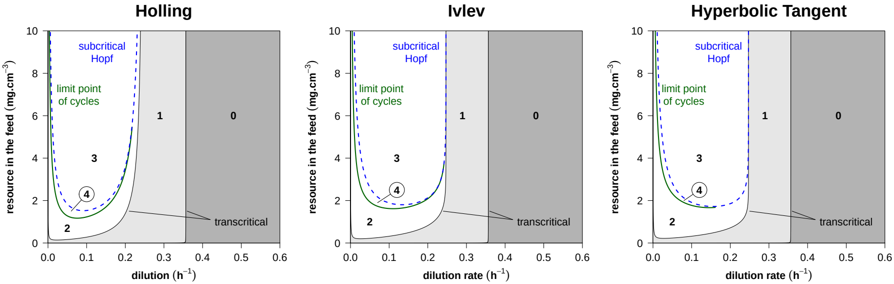








Structural sensitivity in model "DEB"



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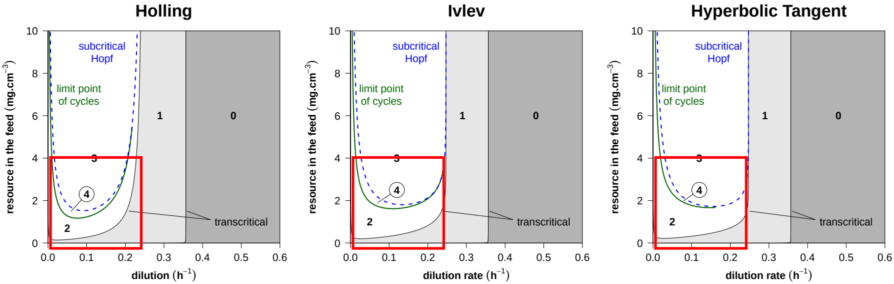

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- same qualitative bifurcations (position $\approx$)
- differences $\subset$ subspace of parameter values

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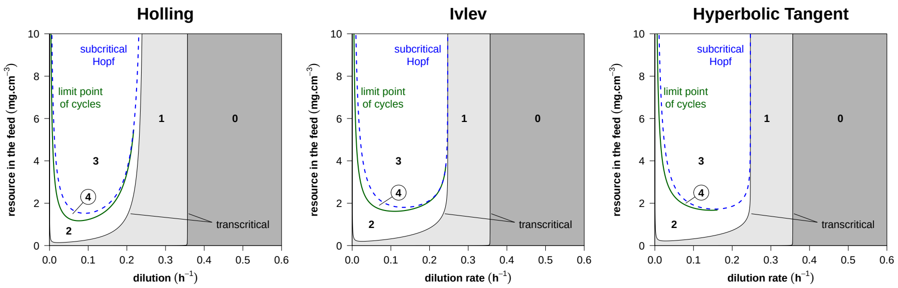
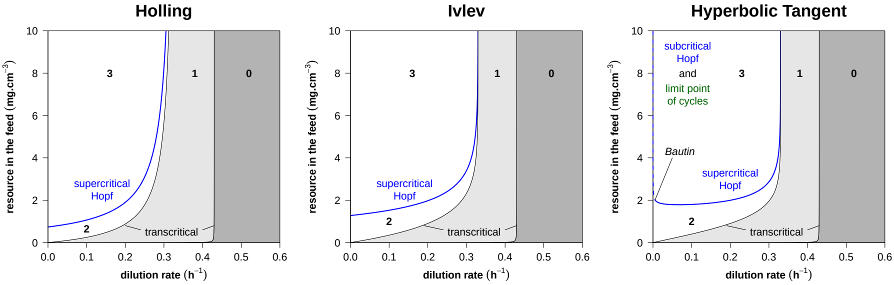


there is sensitivity... yes but

- same qualitative bifurcations (position $\approx$)
- differences $\subset$ subspace of parameter values
- **sensitivity $\ll$ previous models** (no "new" dynamics)
- same conclusion for Droop & Marr-Pirt models (middle complexity)

What about Monod model? (simplest)



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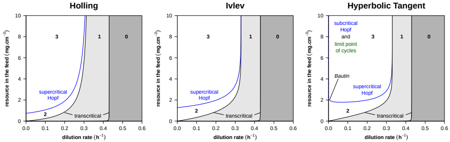
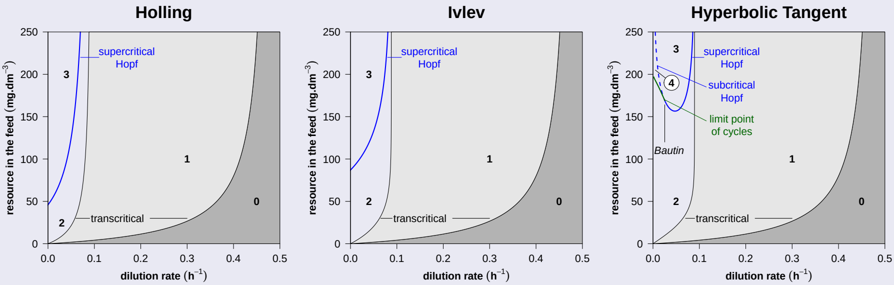


other micro-organisms *(data from Canale et al., 1973)*



A summary of structural sensitivity in predator-prey models

model	maintenance or mortality	explicit reserve	explicit resource
Rosenzweig-MacArthur *(Fussmann & Blasius, 2005)*	x		
Bazykin *(Aldebert et al., 2016)*	x		
Monod			x
Marr-Pirt	x		x
Droop		x	x
"DEB"	x	x	x

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Monod			x
Marr-Pirt	x		x
Droop		x	x
"DEB"	x	x	x

⇓ sensitivity (qualitatively similar bifurcations)

explicit resource + (maintenance OR explicit reserve)

Implications and perspectives

a solution to structural sensivity ?

- general across a wide range of parameter values / species ? (ongoing work)
- does it work within more compex models (food webs...) ?

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- known structural sensitivity to co-limited nutrient uptake and growth (Poggiale *et al.*, 2010), but what is the interplay with predation ?

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Thank you for your attention !